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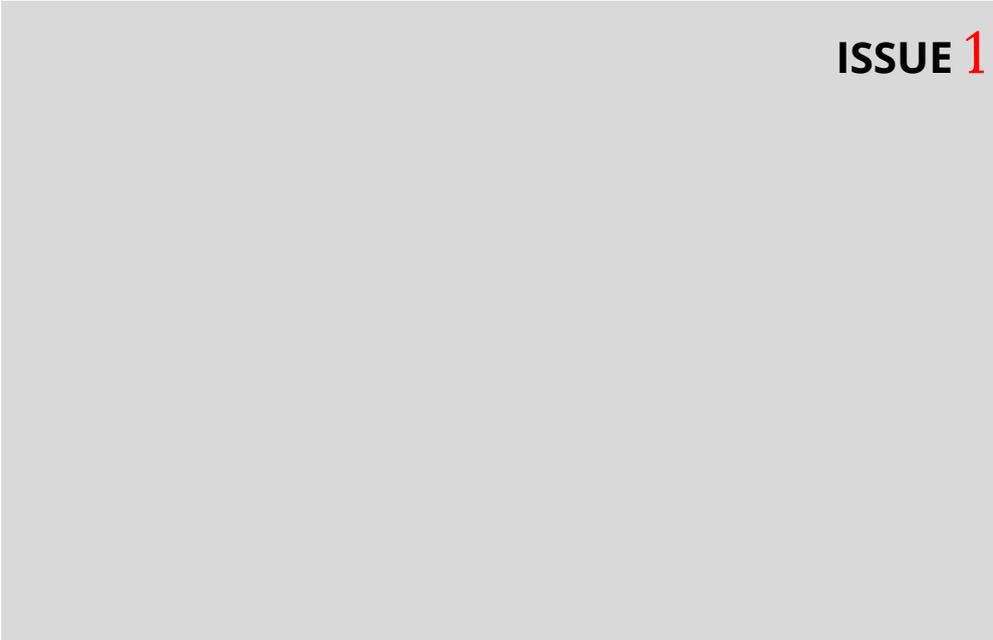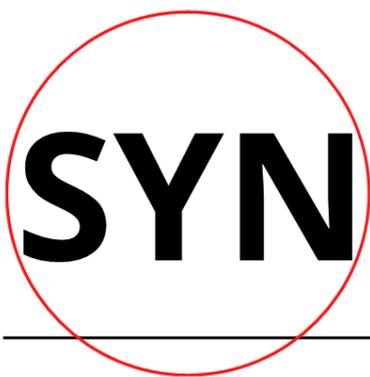
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Non-Existence:
Error and Fiction

List of contributions

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Non-Existence: On the Representability of Nothings

Abstract: The paper discusses the representability of two nothing types from their linguistic and epistemological aspects. It consists of two parts. The first part discusses the representable type of nothing, i.e., *nihil privativum repraesentabile* (abbreviated as NP) and distinguishes its used and mentioned versions. The distinction makes the real nothing cases discernible from the nothing as a general term. Two cases of NP are analyzed. The first one covers Kant's explanations about the identification of absence in experience and the fabrication of non-existent entities such as fictions, dreams, and hallucinations. The second example analyzes a case from Spinoza, where the truth value of existential statements is evaluated, specifically when they appeal to the notions of absence, presence, existence, and non-existence. The second part discusses the irrepresentable type of nothing, i.e., *nihil negativum irrepraesentabile* (abbreviated as NN). Applying the mention and use distinction to NN, the possibility of representing the logically impossible nothings is discussed. There are two cases examined. The first one is Frege's definitions of zero and it is argued that Frege confuses the irrepresentable and representable types of nothing. The second case is Russell's paradox where the Russellian solution is compared with Kant's treatment of NN. The further implications of the analysis of NN are discussed in the context of the truth value gluts, gaps and the truth values of the paradoxical statements. The paper concludes that only the representable nothings are logically possible formal concepts.

Keywords: *Nihil privativum*, *Nihil negativum*, *nothing*, *representability*, *fiction*, *error*

Introduction

There is a myriad of epistemological accounts proposing an answer to how the representation of the existing objects is possible. The fertility in epistemological explanations can be observed in doppelgänger linguistic versions where the main concern is the relation between objects and linguistic terms. Nonetheless, in the case of non-existence, the philosophical investigations have never been as fertile. Arguably, this results from the common wisdom that there is simply nothing to be explained. Contrary to the common wisdom, in what fol-

lows the problem of representability is discussed within the context of two nothing types.[1]

The first part discusses the representability of a type of nothing that Kant coins *nihil privativum repraesentabile*. It is argued that formality[2] is a mode of existence[3] of the thought entities that are conceptually representable. Compared to formality, materiality is the mode of existence that pertains to the entities existing physically. In the context of formality and materiality, the first part focuses on the conditions that allow the conceptual representation of the formal concepts or terms that refer to the materially non-existent entities. There is a linguistic difference between the used and mentioned versions of *nihil privativum*. While the used versions are singular terms designating the absence or non-existence of the particular objects, the mentioned version acts as a general term. Distinguishing these two senses of *nihil privativum*, the first part presents two forms of representing non-existence or absence. Accordingly, the first form regards how we identify the absence of an entity in experience; and the second form concerns the fictitious entities that have no materiality.

In the first part, the focus is not only on the features of fictitious terms and entities but also on how they are fabricated. The first set of examples for *nihil privativum* includes Kant's treatment of made-up terms, fictions, dreams and hallucinations. Although all the *nihil privativum* cases belong to the same class, their fabrication process plays a key role in distinguishing them further

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[1] Despite that I focus on two specific types of nothing within the context of the following debate, the Kantian table of nothing covers four types, namely, (1) *ens rationis*, (2) *nihil privativum*, (3) *ens imaginarium*, and finally (4) *nihil negativum* (see A292/B348-9). Kant gives *noumena* as the example of *ens rationis* (see A290/B347), which can be interpreted as that the role of *ens rationis* is drawing the limits of knowledge. And for *ens imaginarium*, Kant gives the example of the forms space and time (see A291/B347), i.e., the role of *ens imaginarium* seems to be making the unity of experience possible. Unlike these two types, *nihil privativum* and *nihil negativum* are concerned in what follows. As it will get clearer in due course, *nihil privativum* is about the borders of phenomenal reality and *nihil negativum* is about the limits of possible logical thinking. As the paper discusses the possibility of representing absence within the phenomenal borders and representing what is logically impossible, it neglects (1) and (3).

[2] Elsewhere and in a set-theoretical context, I discuss formality and the case of Frege's definiton of zero below. See Birgül (2021).

[3] I beg the pardon of the readers due to the existentialist connotations of the phrase *mode of existence*. The phrase points out the difference between the concrete and abstract objects. However, the term *abstract* can get quite complicated when one considers Plato's ontological framework. Thus, as it will get clearer below, the difference between different modes of existence is simply the difference between the ontological realms or classification of thought entities and material objects.

into sub-types of *nihil privativum*. The second case is from Spinoza's *Ethics* where Spinoza discusses the non-existence of Peter. Spinoza's case clarifies the distinctions between the four terms *absence*, *presence*, *existence*, and *non-existence*. This case provides a ground to discuss the possibility of using these four terms together and attributing them to the same object. As the logical aspect of the second case, the truth values of the statements that combine any two of the four terms are considered.

The second part concerns the irrepresentable type of nothing that Kant coins *nihil negativum irrepraesentabile*. This part begins with an analysis of why *nihil negativum* cases are irrepresentable. It is argued that this is because they do not satisfy one of the necessary conditions of formal existence, namely the logical possibility. The significance of logical possibility is elaborated in a comparison of logical and real oppositions. Like *nihil privativum* cases, the fabrication of the *nihil negativum* cases is crucial, yet from a different aspect. In what follows it is argued that any fabrication of a *nihil negativum* case violates the principle of contradiction. Thus, the principle itself serves as a criterion of classifying the particular nothing-cases under the concepts of *nihil privativum* or *nihil negativum*, and it can be adopted as a tool to detect and avoid the self-contradictory reasoning. It is claimed that the mention and use distinction is applicable to *nihil negativum*. Despite its violation of the principle of contradiction, the distinction makes the representation of the mentioned version of *nihil negativum* as a *nihil privativum* case.

As in the first part, there are two cases analyzed in the context of *nihil negativum*. The first case is Frege's definition of zero, where there is a problem of mistreating non-existence of an extension as if it is an empty extension. The second problem discussed through Frege's case is the confusion of an irrepresentable type of nothing with a representable one. As the second case of *nihil negativum*, Russell's paradox is examined. Accordingly, instead of banning self-reference to avoid the self-referential paradoxes, the denial of the paradoxical concept is endorsed in the light of the analysis of *nihil privativum*. Like the implications of *nihil privativum* in logic, *nihil negativum* has its own implications in the field. Comparing the truth-value gluts and gaps in the context of *nihil negativum*, it is argued that the paradoxical statements take a gap as their truth value.

1. *The Fiction Case:* Nihil Privativum Repraesentabile

In terms of the possibility of conceptual representation, NN and the concept of NP are distinguished by formality. Only the conceptually representable thought entities exist formally. In that regard, unlike NN, the concept of NP is a representable object of thought that exists formally. To specify further, let me begin with Kant's terminology of nothings in the context of formality and materiality:

> A nothing is a merely formal concept, to the extent that the lack of everything material in intuition is found with it. It is thus only a nothing in the material sense <*nihilum in sensu materiali*>, but by all means a something in the formal sense <*in sensu formali*>, ... From this it follows that all concepts belong here for which all empirical intuition is lacking (LM, 29: 962).[4]

The passage indicates two critical points. The first one is that the designatum of the term *nothing* is characterized as non-existent or absent in its material sense. In other words, nothing does not exist. On the other hand, as a term, nothing is classified under the class something in its formal sense, i.e., nothing exists. Notice that both nothings are NP in different senses, i.e., in the material and formal senses. The distinction between these two senses of NP is a distinction between what is essentially linguistic-conceptual and what is essentially metaphysical. Kant warns against confusing these two by distinguishing them: «The mistake is that <*nihil*> is meant one time as negation, another time as a concept» (LM, 29: 815-6). As he contends that «negation is nothing» (A291/B347), the distinction makes it clear that NP is nothing in its material sense, and as an object of thought, it is something in its formal sense.

As is known, when a term is used it refers to the object in question, and when it is mentioned, it refers to a term. Consider the following example: '*Aris-*

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[4] The *Critique of Pure Reason* is quoted according to the German paginations that appear in the first and second editions. Although Kant's other works are quoted by volume and the page number of the *Akademie-Ausgabe*, when available, the Cambridge translations are used (sometimes with minor modifications). The abbreviation key for the titles of the translations is as follows: AHE = *Anthropology, History and Education*, LL = *Lectures on Logic*, LM = *Lectures on Metaphysics*, NF = *Notes and Fragments*, TP = *Theoretical Philosophy, 1755-1770*.

totle' is the name of Aristotle. Without the quotation marks, the term *Aristotle* is used and it designates the great philosopher. When the term is within quote marks, i.e., '*Aristotle*', the term is mentioned and this time it designates not an object but another term: the name of the philosopher. Following the example, let me use NP without quotation marks when it is used and with the marks when it is mentioned. Following the example, it can be formulated that NP, or nothing does not exist in its material sense. Nonetheless, when it is mentioned, 'NP' is the formal concept of nothing, and it exists as a representable object of thought. For Kant, 'NP' is about the negative concepts where there is a lack: «the privative nothing <*nihil privativum*>, which means lack, e.g., light is something *positive*, darkness is something *negative*» (LM, 28: 792). Representability of such negative concepts is possible because despite that the object is not present or existing anymore, the thought of that object «does not contradict the possibility of its mere representation» (LM, 29: 961). Due to this possibility, the first point of the passage can be recapitulated by a reformulation of the famous Parmenidean *dictum*: *Being is, non-being is not*; and the concept of non-being is.

The second indication of the passage regards a further point about 'NP'. Kant defines *nihil privativum* as the «empty data for concepts» (A292/B349), not as empty *datum*. 'NP' is not a concept of some particular empty datum or a particular case of nothing. Instead, it is the concept of nothing in general, or simply it is a general term. The evidence is in the last line of the passage where Kant treats 'NP' as a general term and classifies all the singular terms that refer to some materially non-existent or absent object as 'NP' cases. From this, it can be inferred that we can conceptualize the absence of a particular object X as 'NP$_x$'. Unlike the particular instances of non-existence or absence, 'NP' exhausts all cases where there is a term designating no object of intuition.[5]

After introducing NP and 'NP', let me discuss the Kantian and Spinozist cases. In both cases one can observe that 'NP' is exemplified in two main forms. In the first form, 'NP' is exemplified by any term for an absent object that was

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[5] It might be confusing to construe '*nihil privativum*' as the general term that exhausts all the cases where there are terms with no designatum on the one hand, and as the concept of an empty object on the other. The distinction is between the general and singular usages, i.e., when used in its general sense, '*nihil privativum*' acts as a predicate nominative that classifies the examples as terms with no designatum, e.g., *Pegasus* is a case of '*nihil privativum*'. In its singular usage, '*nihil privativum*' is the concept of an empty object, and *nihil privativum* is the empty object of a concept.

real and present in advance. And in the second form 'NP' is exemplified by made up terms designating no object of intuition. Regarding the first one, Kant contends that «If light were not given to the senses, then one would not be able to represent darkness» (A292/B349). Notice that an object is given first, and as a result of a real opposition, it becomes absent later. When there is a real opposition, «one thing cancels that which is posited by the other; but the consequence is something (*cogitabile*)» (TP, 2: 171). If the real opposition were accompanied by logical opposition, we would not be able to represent the absence of any object. According to Kant's example, «The motive force of a body in one direction and an equal tendency of the same body in the opposite direction do not contradict each other» (TP, 2: 171). For him, the «consequence of such an opposition is rest, which is something (*rapraesentabile*)» (TP, 2: 171). Unlike the first form of 'NP', in the second form the object in question is impossible in reality. This sort of impossibility simply means that these objects cannot be «met with in nature *quoad objectum*» (LM, 29: 961). Kant gives the example of fairy tales, «where there is no possible object corresponds» (LM, 29: 761). That is, when one hears such terms as *the frog prince*, or *the gingerbread man*, it is possible to represent them as imaginary beings because the real impossibility of objects does not bound the limits of representation. Like the fairy tale characters, the made-up terms exemplify the same form of 'NP'. An example of this is the *aether* in physics «that has no reality *intuitu objecti*, but which can be thought without contradiction. ... [It] contains nothing contradictory in thinking it, but it has no existence» (LM, 29: 961).

It is important to consider how one makes up fictions. Unlike the cases of *nihil negativum*, fictions and other similar cases such as hallucinations and dreams are not errors. It is the faculty of imagination where the fictitious entities originate. In fabricating fictitious entities, the function of imagination is reproductive. In other words, it is «a faculty of the derivative presentation of the object (*exhibitio derivativa*), which brings back to the mind an empirical intuition that it had previously» (AHE, 7: 161). This simply means that one cannot imagine a fictitious entity based on some data that has not been given by experience.[6]

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[6] Kant's examples emphasize that not only the objects themselves must be given to imagination by experience but also the objects synthesized by the combination of the previously given ones: «To one who has never seen *red* among the seven colors, we can never make this sensation compre-

Kant clearly states that «the sensations produced by the five senses in their synthesis cannot be made by means of the power of imagination, but must be drawn originally from the faculty of sense» (AHE, 7: 168). To fabricate fictitious entities, imagination acts on two principles: *separation* and *combination*. Kant maintains that by separation,

> I negate something from my concept. ... Through separation only fabricated concepts arise ... A *fictio separando facta* involves a removal of what otherwise is usually combined with a concept. We fabricate much *separando* [;] all novels arise by means of this kind of fiction (LL, 24:262).

As an example, one may imagine a man, a coyote, and then separate or remove their torsos in imagination. This step is separation. Unless one desires to imagine headless or bodiless fictitious entities, the combination should be taken as the second step. According to Kant, by combination, «we fabricate something when we put much together that in experience is never connected. In this way, as already indicated above, novels arise» (LL, 24: 262). Following the same example, one gets the image of *Anubis* when the head of the coyote is combined with a human torso.[7]

Dreams and hallucinations are fabricated by the same principles of separation and combination. On dreams and the origins of their images, Kant writes that «without an outer sense, whose representations we merely repeat and combine in a different way... we would not be able to have any dreams at all» (NF, 18: 310). In dreams one can separate and combine almost any object of intuition including places and different temporal segments into fictitious entities and episodes. Yet, the difference between the fabricated fictitious entities and dreams is that while the former is fabricated consciously, in dreams the entities

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hensible ... Yellow and blue mixed together give green; but the power of imagination would not produce the slightest idea of this color, unless it had *seen* them mixed together» (AHE, 7: 168).

[7] Notice that separation and combination are carried out voluntarily and consciously. This is the main feature of fictions compared to dreams and hallucinations and it can be found in many modern texts. E.g., Spinoza holds that «A Fictitious Being excludes clear and distinct perception, because a man merely according to his fancy–and not unknowingly, as in the case of the false, but knowingly and wittingly– joins together what he wants to join and separates what he wants to separate» (2002b: 178).

are fabricated involuntarily[8]: «An involuntary play of one's images in *sleep* (a state of health) is called *dreaming*» (AHE, 7: 167). Like dreams, hallucinations are involuntary fabrications of fictitious entities, yet with a crucial difference: while asleep, people may occasionally take dreams as if they are real, but the *vice versa* does not happen at least to the people with no mental illnesses. According to Kant, hallucinating is a mental illness where there is a tendency «to accept the play of ideas of inner sense as experiential cognition, although it is only fiction... and accordingly to trick oneself with the intuitions thus formed (dreaming when awake)» (AHE, 7: 161). Hence, one takes the fabricated fictions as if real and this happens while the person is awake. This is not an illness where the faculty of imagination malfunctions but where one loses the track of the difference between the inner and outer senses.

Despite that there is a distinction between non-existence and absence, and between existence and presence, so far, I have used them interchangeably. Regardless of their being distinct concepts, some cases allow using them together while some other cases do not. As we shall see in the following section, the first two combinations are impossible: (a) an entity cannot be simultaneously non-existent and existent, and (b) an entity cannot be simultaneously absent and present. Yet, there are four other combinations among the concepts above, which do not lead to any paradox. These combinations are, (1) an existent entity can be present, (2) an existent entity can be absent, (3) a non-existent entity can be absent, and lastly, (4) a non-existent entity can be present. All four combinations are indicated by the following case that Spinoza (2002a) describes in his *Ethics*:

> In addition (preceding Cor. and Cor. 2 Pr. 16, II), this gives a clear understanding of the difference between the idea, e.g., of Peter which constitutes the essence of Peter's mind, and on the other hand the idea of Peter which is in another man, say Paul. The former directly explicates the essence of Peter's body, and does not involve existence except as long as Peter exists. The latter indicates the constitution of Paul's body rather than the nature of Peter; and so, while that constitution of

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[8] Like the voluntary acts of separation and combination, the involuntariness of these acts in fabricating the contents of dreams and hallucinations can be observed in Spinoza. Unlike fictions, one cannot voluntarily make/suspend judgments about the content of his dreams. Spinoza contends that he does not believe «there is anyone who thinks that while dreaming he has free power to suspend judgment regarding the contents of his dream, and of bringing it about that he should not dream what he dreams that he sees» (2002a: 275).

Paul's body continues to be, Paul's mind will regard Peter as present to him although Peter may not be in existence (257).

Regarding the combination in case (1), unless it is a hallucination or an illusion, we know the existence of the physically present things in our environment. In the Spinozist case, this corresponds to a situation where Peter exists, and he is where Paul can perceive. If the entity is not within the perceivable environment, it may still exist regardless of our inability to perceive it. This is the case (2) where an existent being can be absent. The case (3) is the case where Peter has never existed and thus never been perceived by Paul. Lastly, it is the case (4) where Peter ceases to exist physically, but he can still be remembered or imagined by Paul.

These cases are simple; however, the truth values of the statements can be complicated, especially when compared to the truth values of the statements about the cases of *nihil negativum* in the second part. Let me evaluate the statement 'Peter exists' in accordance with each case. In case (1) the statement is meaningful and *prima facie* true at least in the accounts such as verificationist theory of truth or the correspondence theory of truth. Unlike the case (1), in case (2) we cannot verify the existence of Peter, i.e., the statement can be true or false, but not true and false simultaneously. This is due to the impossibility of the case (a) above, which is an obvious contradiction. Coming to case (3), the statement is not even assertable because one cannot obtain the knowledge of an entity from any source other than experience. Thus, as we shall see in due course, this is a case of *nihil negativum* where neither is there an object nor is there a concept (or in this case, a name that designates the object). The case (4) is an example where a change in status of the physical entity has an impact on the meaning and the truth value of the statements about it. While 'Peter exists' is meaningful and true in a case such as (1), when the entity ceases to exist the statement about it loses its full meaning, at least in the abovementioned truth theories. However, unlike case (3), it is possible to assert the statement because Paul has the knowledge of Peter by experience. Note that an entity cannot be absent and present simultaneously as indicated by the case (b). Thus, Peter is present only as an image or, in Spinozist terms, an idea with no objective validity. In this case, the meaning or the truth value of the statement has been changed

by physical conditions, and Paul re-evaluates the statement when he learns about Peter.

2. *The Error Case*: Nihil Negativum Irrepraesentabile

It is already stated that formality is the mode of existence of the conceptually representable thought objects. Here, the focus is on one of the necessary conditions of formal existence: *logical possibility*. In that regard, a concept exists formally if it satisfies the condition of being logically possible. To compare, an NP case, such as rest, is based on real opposition, while an NN case is based on logical opposition. We can represent the concept of rest because the real opposition cancels the object, not the concept. As Kant states, in an NP case «the concept is indeed possible, but there is no reality there» (LM, 28:544). On the other hand, there is neither reality nor a concept in an NN case because the concept is also cancelled in the logical opposition. One can represent an object in a state of motion or rest. Nonetheless, one cannot represent the same object in both states simultaneously because it exceeds the limits of our conceptual representation: «a body which is both in motion and also, in the very same sense, not in motion, is nothing at all» (TP, 2: 171). Drawing attention to the inner contradiction that cancels the possibility of its representability, Kant maintains that «[to the negative nothing] no *thought* or representation corresponds at all. It is ordinarily so constituted that it involves an inner contradiction in the representation» (LM, 29: 961). This is a domino effect, where the logical impossibility cancels the possibility of representability, and thus the formal existence of the thought entity in question.

The principle of contradiction plays a key role in detecting and avoiding the error in thought. The error in NN cases is the self-contradicting judgment that eventually evolves into a paradox. Unlike the paradoxes such as those of Zeno, NN cases are self-referential paradoxes where a vicious circle is initiated once the erroneous judgment is made. According to Kant, «The point is that two opposing predicates cannot be either affirmed or denied of the same thing. Both are errors» (LM, 28: 793). There is no difference between the simultaneous denial and the simultaneous affirmation of the opposing predicates. As an example, recall the impossible case (a) in the previous part: *an entity cannot be simulta-*

neously non-existent and existent. For Kant, «*Reality* is something; *negation* is nothing» (LM, 28: 543), and once one makes the error of affirming the existence and non-existence of the same thing, it amounts to affirming that the entity is both nothing and something in its material sense. Due to the double negation law, one falls into the same vicious circle if he denies the opposing predicates. According to Kant, the only way to avoid the error is to double-check the reasoning by the principle of contradiction.[9]

There is a seemingly tricky question regarding the discourse and representability of NN. Kant coins NN as the «empty object without a concept» (A292/B348). If there is neither a concept nor an object, there is simply nothing to talk about in both material and formal senses. The question is that how is it still possible to maintain the discourse about NN? For example, how come this part of the article discusses the features of the mere *nothing* as if it is something? To answer the question, let me begin by reminding us that the representational impossibility does not imply the impossibility of the discourse. While logical possibility is one of the conditions that makes the formal existence of the concept possible, it is the mention and use distinction that makes their discourse possible by referring to the terms in their mentioned sense. Defining a *Chimera* as «whose nature involves an open contradiction» (Spinoza 2002b: 178), Spinoza (2002b) presents a brief and precise explanation about the point:

> [A *Chimera is properly called a verbal being.*] Here it should be noted that: 1. Because a chimera is neither in the intellect nor in the imagination, we may properly call it a verbal being, for it can be expressed only in words. For example, we can express a square circle in words, but we cannot in any way imagine it, far less understand it (183).

As is noticed, Spinoza only mentions *Chimera* when he calls it as a *verbal being*. On the other hand, he refers to any chimera in the used sense when he writes that *a chimera is neither in the intellect nor in the imagination*. Once he calls chimera *a verbal being*, the mentioned chimera acts as a general term and can be exemplified by any particular chimera case. When the used and men-

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[9] According to Kant, one can guard himself against contradictions by the principle of contradiction (see LM, 29: 811). Of course, as a field logic advanced a lot after Kant and in evaluating contradictions, there are other alternative positions such as dialetheism. Yet, it seems possible to evaluate them from a Kantian perspective. The details are in due course.

tioned chimeras are transferred in to the Kantian terminology, the used chimera is an NN case, and the mentioned chimera is 'NN'. As a further indication of 'NN', it can be argued that 'NN' is the concept of NN, namely *nothing at all*. Although an NN is mere nothing, 'NN' is a case of 'NP' where there is a term with no object. Due this further indication, it is no surprise that Spinoza (2002b) classifies the mentioned Chimera along with the non-existent things such as «the Fictitious Being and Being of reason» (178), which we cannot identify in experience, yet we are still able to carry out a discourse about. Despite that the formal existence is cancelled by the logical opposition in an NN case, 'NN' is still a formal thought entity that can be represented as a case of 'NP'. If it were otherwise, one would not be able to think about a contradiction without committing to it. Kant affirms the point when he writes, «For formally *<formaliter>* a merely negative thing *<ens mere negativum>* can at least be *thought* without contradiction» (LM, 29:1001).

After this brief sketch of NN and 'NN', let me discuss the first error case, i.e., Frege's definition of zero. His definition is an NN case because he defines zero as a self-contradicting phrase, i.e., «not identical with itself» (Frege 1960: 87). There are no two opposing and distinct predicates in the definition but the law of self-identity that is formulated into self-contradiction through the reflexive pronoun *itself*. Kant formulates the underlying structure by giving the usual definition of NN as «that which at the same time can both be and not be the same thing *<quod idem simul esse et non esse potest>*» (LM, 29: 963). Frege's appeal to this structure and his definition causes two critical problems. The first problem is that despite zero is defined or modeled by the empty set where there is still a set with an empty extension; Frege's definition does not pick out an empty extension, but *nothing at all*. Badiou draws attention to this problem when he writes, «Even the concept 'not identical to itself' could well turn out not to have any existent extension, which is something entirely different from having an empty extension» (2008: 20). This problem is a result of the logical impossibility of the definition *not identical with itself*. Like any NN case, *not identical with itself* «is a thought that cancels itself, a concept that collapses by inner contradiction» (LM, 29: 962). To specify, it collapses into *nothing at all*.

The second problem is about the countless possible predicates that can be used in generating NN cases. Frege is aware that any other quality or a pred-

icate can be used in generating a contradiction. He lists *square circle* and *wooden iron*[10] among the possible candidates. We see the same or similar examples being introduced in Kant's works as NN cases, e.g., «four cornered circle» (LM, 29: 761), or «bright darkness» (LM, 29: 792). Frege seems to be making a point when he explains why he favors *not identical with itself* to the other candidates. According to him, the preferred definition of zero is different from the other contradictions in being provable «on purely logical grounds» (Frege 1960: 88). To clarify, Frege writes that whether a definition involves a contradiction is not always evident at first sight and the demand of designing a definition under which no object falls is satisfied by his *not identical with itself* (see Frege 1960: 87). In other words, he favors an analytic statement over the synthetic ones because while the contradiction in the former is evident, the latter may require further investigation to see if there is a contradiction. Nevertheless, Frege makes a mistake by believing that the self-contradicting definitions do yield concept under which no object falls: «I could have used for the definition of nought any other concept under which no object falls» (1960: 88). Frege mistakes an NN case for 'NP'. Regardless of the analyticity or syntheticity of its components, no logical contradiction produces an empty extension. Due to this, his definition does not serve as a definition of a thought entity, but nothing at all.

The second error case is Russell's paradox. In formulating the paradox, Russell does not only points out the assumption that eventually leads to the paradox but also offers a solution that Kant's treatment of NN has already pointed out. The formulation is as follows:

> A class-concept may or may not be a term of its own extension. "Class-concept which is not a term of its own extension" appears to be a class-concept. But if it is a term of its own extension, it is a class-concept which is not a term of its own extension, and *vice versâ*. Thus we must conclude, against appearances, that "class-concept which is not a term of its own extension" is not a class-concept (Russell 1903: 103).

The whole paradox rests on the assumption that the *class-concept which is not a term of its own extension* is (seemingly) a class concept. The self-reference follows naturally from the assumption because it falsely assumes that there

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[10] See Frege (1960: 87-88).

is a class-concept with an extension. To paraphrase in the symbolic language, the statement $(S \in S) \rightarrow \neg (S \in S)$ and its reverse $(S \in S) \rightarrow \neg (S \in S)$ hold. By the biconditional operator they can be combined into the famous self-contradictory statement $(S \in S) \leftrightarrow \neg (S \in S)$. To avoid the paradox, one can follow any of the following options: (1) banning self-reference and (2) denying that the term *class-concept which is not a term of its own extension* is a class-concept. When compared, I do not endorse the option (1) because it changes the term completely by amputating the part *its own*. (1) is not feasible because it is not the new non-self-referential term we are concerned with. Coming to the option (2), the term denies the possibility of the class-concept that is both a term of its own extension and not a term of its own extension. This option is favored by Russell and can be supported by an appeal to Kant's treatment of NN. Consider Kant's following analysis of *bright darkness*. As he contends, once the logically opposing thoughts are set together, as in the final formulation of Russell's paradox with a biconditional, there is simply *nothing at all* or *no concept* to think about:

> I think nothing at all when I think the affirmative and negative at the same time, e.g., bright darkness. Here I cannot think darkness, because I think brightness, and not brightness, because I think darkness – thus nothing at all. If I think of two opposed things, then I have two thoughts; if I set them together, then I think nothing at all. (LM, 28:792).

The implications of the points discussed through the two error cases can be observed in comparably recent debates in logic. The concern is about the existence of paradoxes and the truth values of the paradoxical statements. Before getting into the details, let me introduce the truth value gluts and the truth value gaps. A sentence has a truth value glut if its truth value is interpreted as both true and false. Such a sentence is called a *dialetheia* and the existence of dialetheias is defended by dialetheists, most eminently by Priest and Routley.[11] In search for an answer to this question, a proposal by Wittgenstein is worthy of consideration. This proposal is one of the influential motivations of dialetheism. It reads as follows:

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[11] See Priest (2006) and Priest *et al.* (1989).

> Why should Russell's contradiction not be conceived of as something supra-propositional, something that towers above the propositions and looks in both directions like a Janus head? ... The proposition that contradicts itself would stand like a monument (with a Janus head) over the propositions of logic (Wittgenstein 1978: 256 (III. 59)).

Considering the differences between an NP and an NN, *Janus head* is an NP, a fictitious entity. If one follows Wittgenstein and construes the third value as a supra-propositional truth-value glut, this would be mistaking NP for an NN. The reason is that the third value as both true and false is an NN. While *Janus head* is a representable NP, the third value is only an irrepresentable NN, which is why one cannot use them interchangeably. Hence, it can be claimed that Wittgenstein's proposal is an example of conceptual confusion. There is no doubt that the paraconsistent logics are legitimate, and it is possible to interpret the third value from a Kantian point of view. From this perspective one can support the view that the paradoxical sentences lack a truth value that we can represent or comprehend. In other words, they have a truth-value gap because their truth value exceeds the limits of conceptual representation. From a Kantian perspective, neither can one fill that gap nor can defend such an attempt.

Conclusion

The conceptual representation of the different concepts of nothing seems to be a confusing issue. That having been said, we can comprehend and represent not only the particular cases of absence and non-existence but also the concepts absence and non-existence in general. However, there are always some limits of conceptual representation. The mention and use distinction plays a key role in distinguishing the representable and irrepresentable types of nothing from each other. Accordingly, a concept of nothing is representable if mentioning or using it as a concept is logically possible. The logical possibility is one of the necessary conditions of formal existence of the thought entities. It can be concluded that a *nihil privativum* case is a concept with no object. As the mentioned version, 'NP' is a general term from its linguistic aspect, and a representable concept from its epistemological aspect. *Nihil negativum* is mere nothing when used. Nonetheless, when mentioned, it is a representable and formal object of thought and an example for 'NP'.

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